**Título: An implicit enumerative algorithm for solving the min-degree constrained minimum spanning tree problem**

**Enquadramento:**

Given a weighted undirected graph G=(V,E) with positive costs on the edges and a positive constant d, the min-degree constrained minimum spanning tree problem is to find a minimum cost spanning tree T of G where each node I of T either has degree at least d or is a leaf node. Thus, the goal is to find a spanning tree T where all its non-leaf nodes (or central nodes) have degree at least d. These central nodes are not given in advance, and the problem is to find the least cost spanning tree with the mentioned lower bounding degree propriety on the nodes.

This problem was first introduced in 2006, being denoted by md-MST. When $3 \leq d \leq \lfloor |V| /2 \rfloor$, the problem was shown to be NP-hard (see, [1] and [2]). This means that it is unlikely that a polynomial time algorithm exists for d in the mention ed range.

Some bounding properties for the number of central nodes were discussed in [1]. In addition, a large number of formulations were proposed for the md-MST problem (see, [1], [3] and [5]). These formulations were tested and discussed within branch-and-bound and branch-and-cut frameworks. Additional valid constraints were also proposed in order to strengthen the associated linear programming relaxation versions.

The only heuristic approaches for the md-MST were introduced in [4]. In this paper, the authors proposed a greedy algorithm and used it for building Variable Neighborhood Search (VNS) and Enhanced Second Order based algorithms for the problem.

There is still a large amount of work to do on the md-MST. One of the areas that still require additional effort involves better techniques for efficiently solving the problem. In effect, and from our best knowledge, there is no implicit enumerative algorithm for the md-MST, which focuses the main concern of the present work. It is also important to stress the closely related "Degree-Constrained Minimum Spanning Tree (DCMST) problem, being discussed in the literature since long. Compared with the md-MST, the DCMST involves an upper bounding degree constraint in all nodes of the tree, instead.

**Objectivos:**

The student should develop an implicit enumerative algorithm for the md-MST and conduct a number of computational tests in order to compare the performance of the algorithm with other exact methodologies in the literature. For this goal, the student must analyze, implement and discuss known lower and upper bounding techniques for the md-MST, using some of the properties and heuristics previously published, or others being discovered.

**Plano de trabalhos**

- Analyze lower and upper bounding methods for the md-MST;
- Implement an implicit enumerative algorithm for the md-MST;
- Conduct computational tests on benchmark instances and compare the results with other alternative exact methods in the literature.

**Pré-requisitos:**

The student should be proficient in procedural programming (C or C++ or Java), data structures and algorithms, in particular, an above-average grade in *Algoritmos e Estrutura de Dados* and *Laboratório de Avançada Programação*.

**Orientação:**

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## Referências


[1] Almeida A M, Martins P, Souza M C.. Min-degree constrained minimum spanning tree problem: complexity, properties, and formulations. International Transactions in Operational Research 2012; 19: 323-352.

[2] Almeida A M, Martins P, Souza M C. md-MST is NP-hard for d □ 3. Electronic Notes in Discrete Mathematics 2010; 36: 9-15.

[3] Martinez L C, da Cunha A S. The min-degree constrained spanning tree problem: Formulations and Branch-and-cut algorithm. Discrete Applied Mathematics 2012, in press.

[4] Martins P, Souza M C.. VNS and second order heuristics for the min-degree constrained spanning tree problem. Computers & Operations Research 2009; 36: 2969-2982.

[5] Akgün I, Tansel B Ç. Min-degree constrained spanning tree problem: New formulations via Miller-Tucker-Zemlin constraints. Computers & Operations Research 2010; 37: 72-82.